Universality class for domain growth in random magnets

1991 J. Phys. A: Math. Gen. 24 L1185

(http://iopscience.iop.org/0305-4470/24/19/010)

J. Phys. A: Math. Gen. 24 (1991) L1185–L1191. Printed in the UK

LETTER TO THE EDITOR

Universality class for domain growth in random magnets

A J Bray and K Humayun
Department of Theoretical Physics, University of Manchester, Manchester M13 9PL, UK

Received 7 June 1991

Abstract. A 2D Ising model with random ferromagnetic bonds is studied by Monte Carlo simulation following a quench from $T = \infty$ to $T < T_c$. The data are consistent with the theoretical prediction $x = 4$. The exponent $\bar{\lambda}$, defined by $\langle S_i(0)S_i(t)\rangle \sim L(t)^{-\bar{\lambda}}$, and the scaling functions for the spatial correlations, are very close to those of the pure system, suggesting that pure and random systems belong to the same universality class.

The kinetics of domain growth following a quench from the disordered to the ordered phase has been a focus of considerable attention for some time [1]. It is known that the ordering dynamics are governed principally by the conservation property of the order parameter. A conserved order parameter gives rise to 'model B' dynamics, which describes phase separation or 'spinodal decomposition' in alloys. Here we are interested in the kinetics of a scalar, non-conserved order parameter ('model A' dynamics), appropriate for the order–disorder transitions observed in many binary alloy systems.

The kinetics of a non-conserved 2D Ising model without impurities, following a quench from high temperature into the ordered phase, are well understood [2,3]. A scaling regime is entered for sufficiently long times during which the growth of order is characterized by one length scale, the *domain scale* $L(t)$. The prediction that $L(t)$ scales as $t^{1/2}$, independent of spatial dimension d, has been amply confirmed by both analytic [2–4] and numerical [5–8] work as well as by experimental studies [2] on ordering alloys such as Fe–Al and Cu–Au. However, the effect of quenched impurities on the kinetics of domain growth is less well understood. Predictions have been made for the growth law of $L(t)$ for the random-bond Ising ferromagnet (RBIF) using scaling arguments based on the energetics of a single interface [9]. These suggest that asymptotically the domains grow logarithmically with time, $L(t) \sim (\ln t)^x$, where $x = (2 - \zeta)/\chi$ is related to the exponents χ and ζ describing the scale-dependence of energy barriers in the system, and of interfacial roughening due to the disorder, respectively. Previous simulation studies confirm the slow dynamics but the exponent x has never been accurately determined. The results of Grest and Srolovitz [10] on the disordered Ising ferromagnet did suggest a logarithmic form for $L(t)$, but they were unable to distinguish a simple $\ln t$ behaviour from a power of $\ln t$. Subsequent simulations [11] have confirmed the $(\ln t)^x$ form, but again a precise determination of x has not proved possible. In fact a direct fit of $L(t)$ to $(\ln t)^x$ generally gives x significantly smaller than the theoretical prediction (for $d = 2$) of 4. Since $\ln t$ is never particularly large, however, it is important in practice to include a scale time

0305-4470/91/191185+07$03.50

t_0 in the fit, i.e. to fit to $[\ln(t/t_0)^x]$. With this refinement, we find results consistent with $x = 4$.

In previous studies $L(t)$ has been measured indirectly. This letter is, to our knowledge, the first time that $L(t)$ for a disordered system has been obtained from a direct measurement of the equal-time spin–spin correlation function $C(r, t, t)$, through the scaling property $C(r, t, t) = f_e(r/L(t))$. We find that scaling is well satisfied for $L(t) = L_0 + [A\ln(t/t_0)]^4$ for a suitable t_0, where the offset L_0 is a correction to scaling. However, our main purpose here is not to determine the precise form of the domain growth law, which would require orders of magnitude more CPU time than we have used, but rather to establish the *universality class* for domain growth in disordered systems. To this end we compute the exponent $\bar{\lambda}$, which describes the correlation of the order parameter field with the initial conditions [12,13]. Specifically, the autocorrelation function $A(t) \equiv \langle S_i(0)S_i(t)\rangle \sim L(t)^{-\bar{\lambda}}$. We find a $\bar{\lambda}$ close to that of the pure system. Furthermore, the *scaling function* $f_e(x)$ is indistinguishable from that of the pure system, confirming a conjecture made in [12]. The scaling function for the spatial correlation with the initial condition is also very close to the that of the pure system. Together, these results strongly suggest that the *universality class* for domain growth in disordered systems is the same as that of pure systems.

The equal-time correlation function, $C(r, t, t) = \langle\phi(x, t)\phi(x + r, t)\rangle$, where $\phi(r, t)$ is the (scalar) order parameter field and the angle brackets indicate averages over disorder, thermal noise and initial conditions, is a central quantity in the study of domain growth. It is generically found to exhibit the scaling form [1]

$$C(r, t, t) = f_e(r/L(t)). \tag{1}$$

Correlations of the order parameter at two different times are also of interest, since these are described by the non-trivial dynamic exponent $\bar{\lambda}$ [12,13]. We define $C(r, 0, t) = \langle\phi(x, 0)\phi(x + r, t)\rangle$, the spatial correlation with the initial condition. General scaling arguments [13,14] suggest

$$C(r, 0, t) = L(t)^{-\bar{\lambda}} f(r/L(t)). \tag{2}$$

This letter is organized as follows. First, we contrast the dynamics of the RBIF to those of the pure system and then recall the scaling arguments that lead to the predicted logarithmic dependence of $L(t)$ on time. We then give details of our simulations and present the results. In particular, the scaling functions $f_e(x)$ and $f(x)$ in (1) and (2) are obtained for the first time in a random system. We find that both they, and the exponent $\bar{\lambda}$, are very similar to the pure system results obtained in previous studies, i.e. our data support the idea that *the random system is in the same universality class as the pure system.*

Consider an Ising ferromagnet with random exchange couplings, quenched from the high-temperature phase to a temperature $T < T_c$. The system will order in a manner qualitatively similar to the pure system, i.e. it will order locally with domain walls separating regions ('domains') of predominantly up and down spins. The average linear domain size $L(t)$ grows with time due to domain wall motion driven by the curvature of the walls [2,3]. Disorder, however, breaks the translational symmetry of the system, and the domain walls tend to be pinned in certain favourable locations where the exchange couplings are weaker than average. Late-stage growth requires thermally activated motion of the domain walls over scale-dependent energy barriers.

The 'standard argument' for $L(t)$ may be paraphrased as follows. A single domain wall in equilibrium, imposed by, for example, applying antiperiodic boundary conditions in one direction, is characterized by a 'roughening exponent' ζ such that the typical transverse displacement of the wall, due to the disorder, over a length L is of order L^{ζ}, and by an exponent χ such that the typical sample-to-sample fluctuation in the energy of the wall around its mean value varies as L^{χ}. These exponents are related by the scaling law [9] $\chi = 2\zeta + d - 3$. For $d = 2$ they have the values [9] $\zeta = 2/3$, $\chi = 1/3$. To apply these concepts to non-equilibrium domain growth, one makes the further assumption that the energy barriers between metastable positions of a piece of wall of length L also scale as L^{χ}. Naively, one might assume that the appropriate lengthscale to determine the barriers to domain growth at time t would be the domain scale $L(t)$ itself. This would imply, through simple Arrhenius activation, the relation $\ln t \sim L(t)^{\chi}/T$, or $L(t) \propto (\ln t)^{1/\chi}$, i.e. $L(t) \sim (\ln t)^3$ for $d = 2$. However, Villain [15] has argued that the walls can move in shorter sections of length l, where l is the minimal lengthscale on which the domain walls 'notice' their curvature, i.e. the disorder roughening $\sim l^{\zeta}$ should be comparable to the distortion $\sim l^2/L(t)$ due to the curvature of the wall (with typical radius of curvature $L(t)$). This gives $l \sim L(t)^{1/(2-\zeta)}$, an activation barrier of order $l^{\chi} \sim L(t)^{\chi/(2-\zeta)}$ and a domain growth law $L(t) \sim (\ln t)^x$ with $x = (2 - \zeta)/\chi$. An equivalent argument in terms of driving forces (due to curvature) and pinning forces (due to disorder) has been given by Nattermann [16]. For $d = 2$ the prediction is $L(t) \sim (\ln t)^4$.

The main goals of this work were to test this prediction through Monte Carlo simulations, and to determine the universality class for domain growth in random systems through measurements of the exponent $\bar{\lambda}$ and the scaling functions. The Hamiltonian is the conventional Ising Hamiltonian $H = -\sum_{(i,j)} J_{ij} S_i S_j$, where the sum is over nearest-neighbour pairs and $J_{ij} > 0$. Simulations were performed for lattice sizes of up to $N = 600^2$ spins, with periodic boundary conditions. Data for smaller systems show that the results presented here for $N = 600^2$ are not significantly finite-size affected. The spins are initially given a random configuration (corresponding to a $T = \infty$ state) and then evolved at a temperature $T < T_c$ using a standard heat bath algorithm, vectorized by sequential updating of each sublattice in turn. The results were averaged over an ensemble of 84 independently generated initial configurations (of spins and bonds), with the final time measurement being at 15 000 MCS. (1 MCS means one update of both sublattices).

Some preliminary studies using different disorder distributions and quench temperatures were made in order to choose parameters giving a reasonably large scaling regime in the simulation time available. For very weak disorder, scaling characteristic of the pure system ($L(t) \sim t^{1/2}$) persists until late times, whereas for very strong disorder the system rapidly becomes frozen up, with very slow domain growth. The data presented below were obtained with a final quench temperature $T_f = 0.35\, T_c(1)$, where $T_c(1)$ is the critical temperature for a *pure* 2D Ising model with $J_{ij} = 1$. The lattice bonds were distributed uniformly in the range $0.4 \leqslant J_{ij} \leqslant 1.6$. Note that $T_f < T_c(0.4)$, so T_f is definitely below T_c of the random system. In most previous studies [10, 11] disorder has been introduced through dilution. To generate reasonably strong disorder, however, it is necessary to go quite close to the percolation threshold, which introduces additional complications associated with the fractal geometry on length scales shorter than the percolative correlation length. These unwanted complications are avoided by using a continuous distribution of bond strengths.

Data for the equal-time corelation function $C(r, t, t) = \langle S_i(t) S_j(t) \rangle$, (where

$\langle\cdots\rangle$ indicates a simultaneous average over initial conditions, thermal noise and quenched disorder) are shown in figure 1 where, as elsewhere in this letter (unless stated otherwise) the errors are smaller than the symbols. The abscissa in the plot is the scaling variable $r/L(t)$, where $L(t)$ was fixed by the condition $C(L(t),t,t) = 1/2$. The excellent collapse of the data on to a scaling curve (times $\geqslant 1000$ are shown) confirms the scaling form (1). The fact that the scaling function in figure 1 extrapolates to a value close to unity for $r/L(t) \to 0$ shows that the equilibrium magnetization is almost saturated at this temperature. Included on the plot are the $T = 0$ data [8] for the pure system at $t = 640$ MCS, with $L(t)$ defined the same way. These data fit perfectly onto the same scaling curve.

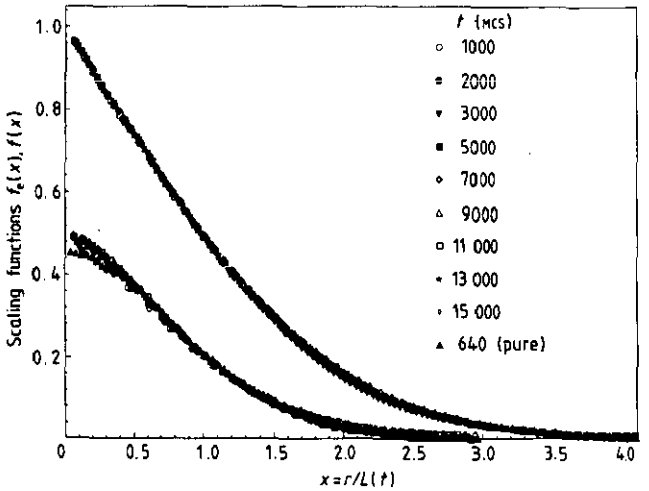

Figure 1. Scaling plots for the 2D RBIF quenched from $T = \infty$ to $T = 0.35\,T_c$ (1). The upper data set is the equal-time correlation function, $C(r,t,t)$, calculated for sites i and j separated by r lattice spacings along a lattice direction. The data represent an average of 84 histories of a 600^2 system, for times up to 15 000 MCS. The domain size $L(t)$ was determined from $C(L(t),t,t) = 1/2$. The lower data set give the scaling plot for the spatial correlation with the initial spin configuration, plotted as $L(t)^{\bar{\lambda}}C(r,0,t)$ versus $r/L(t)$, with $\bar{\lambda} = 1.24$. Data for the pure system at $T = 0$ and $t = 640$ MCS are included in both data sets for comparison.

We have tried to determine the logarithmic domain growth exponent x by fitting $L(t)$ to $[\ln(t/t_0)]^x$. We found it impossible, however, to determine x with any precision due to the limited dynamic range available. A direct fit to $(\ln t)^x$ (i.e. $t_0 = 1$) is consistent with the data between 2000 and 15000 MCS for $x = 3$, but so are other reasonable values of x for $t_0 \neq 1$: there is no *a priori* reason to choose $t_0 = 1$. Instead, therefore, we *assume* that $x = 4$ is correct. We can then fit all the data to the form

$$L(t) = L_0 + [A\ln(t/t_0)]^4. \tag{3}$$

The 'offset' L_0 is an attempt to incorporate the leading 'correction to scaling', which is expected [12] to be of relative order $1/L(t)$. To determine L_0 and t_0 we plot $(L - L_0)^{1/4}$ versus $\ln t$ and choose L_0 to give the best straight line. This gives $L_0 = 4.49$, $A = 0.2227$ and, from the intercept with the abscissa, $t_0 = 0.4668$. The data for $L(t)$ are presented in figure 2. The straight-line fit shown was used

for $L(t)$ in the scaling plots of figure 1. It should be emphasized, however, that the quality of the scaling plots obtained, and the conclusions concerning universality classes, are not dependent on this particular way of fitting $L(t)$.

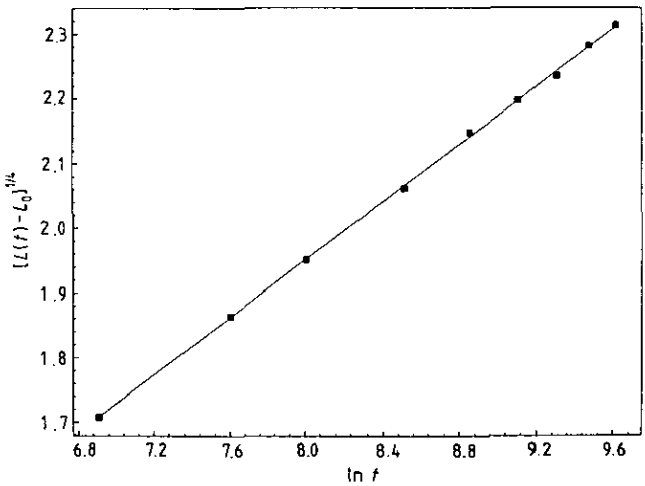

Figure 2. Time-dependence of the domain size $L(t)$, fitted to (3) with $L_0 = 4.49$. The straight line shows the best fit to the data.

In figure 1 we also present the data for the scaling function $f(r,t)$, defined in (2), plotted as $L(t)^{\bar{\lambda}}\langle S_i(0)S_j(t)\rangle$ versus $r/L(t)$, using the pure system value [8] $\bar{\lambda} \simeq 1.24$. The data collapse is very satisfactory. Again, the $T = 0$ pure system data [17] at $t = 640$ MCS are included for comparison. The agreement is generally excellent except for small $r/L(t)$ where both pure and random systems fail to scale perfectly, for reasons which we now discuss.

In previous studies of *pure* systems the dynamic exponent $\bar{\lambda}$ has been measured directly through the auto-correlation function, $A(t) = \langle S_i(t)S_i(0)\rangle$. Note that $A(t)$ is simply the spatial correlation with the initial condition, $C(r,0,t)$, with $r = 0$. As in the corresponding results for the pure system [8,12], there is some curvature in the data for $A(t)$. Therefore, the data were analysed using a procedure similar to that employed in studies of spinodal decomposition [18], and in pure system studies [8,12]: an effective exponent is defined via

$$\bar{\lambda}_{\text{eff}} = -\log_{10}[A(t)/A(2t)]/\log_{10}[L(t)/L(2t)]. \tag{4}$$

This effective exponent is shown versus $1/L(t)$ in figure 3. The errors in this plot are comparable to the scatter of the data points. It is plausible that the deviation of $\bar{\lambda}_{\text{eff}}(t)$ from the asymptotic $\bar{\lambda}$ should vanish as the length-to-area ratio of the domains: $\bar{\lambda}_{\text{eff}}(t) - \bar{\lambda} \sim 1/L(t)$. A linear extrapolation of the data to $1/L(t) = 0$ gives $\bar{\lambda} = 1.26 \pm .02$, consistent with the pure system estimate of 1.24. At the latest time reached in the simulations, however, $\bar{\lambda}_{\text{eff}}(t) \simeq 1.17$, which accounts for the small departures from scaling evident in figure 1.

We have noted that the scaling functions for $C(r,t,t)$ and $C(r,0,t)$ for the random-bond system are virtually identical to corresponding functions for the pure Ising system [8, 17]. In particular, $C(r,t,t)$ is linear in r for small r, i.e. $C(r,t,t) \simeq$

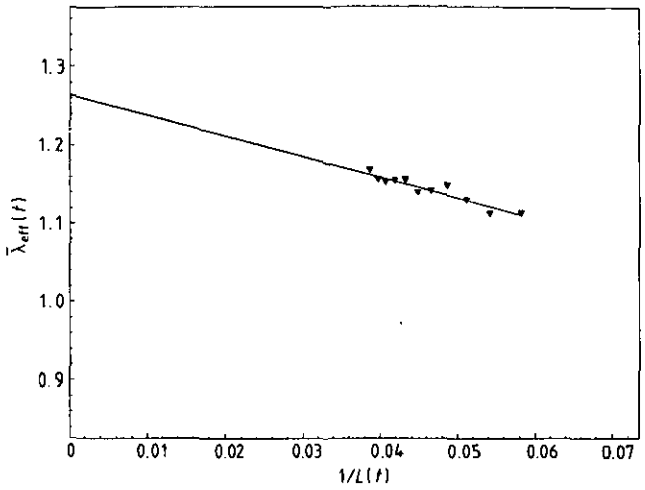

Figure 3. Effective exponent $\bar{\lambda}_{\rm eff}$, defined in the text, as a function of the reciprocal of the domain size $L(t)$. The data represent an average of 84 histories. Each data point further represents the average over a length of time of 500 MCS (a 'smoothing' procedure designed to reduce statistical fluctuations). The continuous curve is the best straight line through the data.

$1 -$ constant $(r/L(t))$ for $r \ll L(t)$, consistent with the familiar 'Porod's law' of pure systems [19]. At first sight this is surprising, as the derivation of Porod's law assumes smooth domain walls whereas in the random-bond system the walls are roughened by the disorder. For rough walls, the structure factor should behave as $S_k(t,t) \sim k^{-(d+\zeta)}$ for $kb \gg 1$, where b is a 'crossover length' defined such that the width of the wall at length scale L due to disorder roughening is $w \simeq b(L/b)^{\zeta}$ [20]. Since $w/L \simeq (b/L)^{(1-\zeta)}$, and $\zeta < 1$, the walls are 'rough' ($w \gg L$) for $L \ll b$, but effectively smooth ($w \ll L$) for $L \gg b$. In the latter regime, which includes the late-stage scaling region since b is time-independent, the Porod law $S_k(t,t) \sim k^{-(d+1)}$ is recovered [20]. (Porod's law survives thermal roughening of the domain walls for the same reason—the walls are smooth on the scale of the domain size.) This line of argument also explains why one expects pure and random systems to be in the same universality class for phase ordering. From a renormalization group viewpoint, the disorder is irrelevant at the strong coupling fixed point ('ferromagnetic sink') which drives the domain growth: the fluctuations in the coarse-grained Hamiltonian, scaling as L^{χ}, are asymptotically negligible compared to the mean, which scales as L^{d-1}. (The result $\chi < d-1$ follows from the scaling law $\chi = 2\zeta + d - 3$ and the inequality $\zeta < 1$ which holds for any system above its lower critical dimension.) However, there is a non-trivial renormalization of the kinetic coefficient on scales up to the 'Villain length' $l \sim L(t)^{1/(2-\zeta)} \ll L(t)$, which leads to the slower domain growth in the disordered system. On scales larger than l, the growth is curvature driven, as in the pure system, and should give rise to the same scaling functions.

In summary, we have studied the growth of order in a random bond 2D Ising system, following a quench from high temperature into the ordered phase. The domain size grows logarithmically, $L(t) \sim (\ln t)^x$, but to pin down the exponent x would require orders of magnitude more computer time. Instead we have devoted most attention to the scaling functions and to the exponent $\bar{\lambda}$ that enters the scaling form for the two-time correlation function. These results strongly support the idea

that pure and random systems are in the same universality class.

After the manuscript was complete we received a preprint from Puri *et al* [21], who have studied domain growth in random magnets using a cell dynamical approach. Their domain size results are consistent with $(\ln t)^4$ growth, and the equal-time scaling function is the same as for the pure system, consistent with our findings. They did not study two-time correlations.

AB thanks M A Moore and A P Young for helpful discussions, M Grant and G S Grest for useful correspondence, and S Puri for a preprint of [21]. KH thanks the SERC for a Research Studentship.

References

[1] For reviews see, e.g., Gunton J D, San Miguel M and Sahni P S 1983 *Phase Transitions and Critical Phenomena* vol 8, ed C Domb and J L Lebowitz (New York: Academic) p 2607
Furukawa H 1985 *Adv. Phys.* **34** 703
Binder K 1987 *Rep. Prog. Phys.* **50** 783
[2] Allen S M and Cahn J W 1979 *Acta Metall.* **27** 1085
[3] Lifshitz I M 1962 *Zh. Eksp. Teor. Fiz.* **42** 1354 (Engl. transl. 1962 *Sov. Phys.-JETP* **15** 939)
[4] Kawasaki K, Yalabik M C and Gunton J D 1978 *Phys. Rev.* A **17** 455
Ohta T, Jasnow D and Kawasaki K 1982 *Phys. Rev. Lett.* **49** 1223
Grant M and Gunton J D 1983 *Phys. Rev.* B **28** 5496
Mazenko G F and Valls O T 1983 *Phys. Rev.* B **27** 6811
[5] Sahni P S, Grest G S and Safran S A 1983 *Phys. Rev. Lett.* **50** 60
Safran S A, Sahni P S and Grest G S 1983 *Phys. Rev.* B **28** 2693
[6] Phani M K, Lebowitz J L, Kalos M H and Penrose O 1980 *Phys. Rev. Lett.* **45** 366
Sahni P, Dee G, Gunton J D, Phani M K, Lebowitz J L and Kalos M H 1981 *Phys. Rev.* B **24** 410
[7] Gawlinski E T, Grant M, Gunton J D and Kaski K 1985 *Phys. Rev.* B **31** 281
Vinals J, Grant M, San Miguel M, Gunton J D and Gawlinski E T 1985 *Phys. Rev. Lett.* **54** 1264
[8] Hymayun K and Bray A J 1991 *J. Phys. A: Math. Gen.* **24** 1915
[9] Huse D A and Henley C L 1985 *Phys. Rev. Lett.* **54** 2708
Huse D A, Henley C L and Fisher D S 1985 *Phys. Rev. Lett.* **55** 2924
[10] Grest G S and Srolovitz D J 1985 *Phys. Rev.* B **32** 3014
[11] Chowdhury D, Grant M and Gunton J D 1987 *Phys. Rev.* B **35** 6792
Chowdhury D and Kumar S 1987 *J. Stat. Phys.* **49** 855
Oh J H and Choi D I 1986 *Phys. Rev.* B **33** 3448
Chowdhury D 1990 *J. Physique* **51** 2681
[12] Fisher D S and Huse D A 1988 *Phys. Rev.* B **38** 373
[13] Newman T J and Bray A J 1990 *J. Phys. A: Math. Gen.* **23** L279; 4491
[14] Furukawa H 1989 *J. Phys. Soc. Japan* **58** 216; 1989 *Phys. Rev.* B **40** 2341
[15] Villain J 1984 *Phys. Rev. Lett.* **52** 1543
[16] Nattermann T 1985 *Phys. Status Solidi* b **132** 125
[17] Humayan K and Bray A J 1991 to be published
[18] Huse D A 1986 *Phys. Rev.* B **34** 7845
[19] Porod G 1982 *Small-Angle X-ray Scattering* ed O Glatter and O Kratsky (New York: Academic)
Debye P, Anderson H R and Brumberger H 1957 *J. Appl. Phys.* **28** 679
[20] Wong P-z 1985 *Phys. Rev.* B **32** 7417
Wong P-z and Bray A J 1988 *Phys. Rev.* B **37** 7751
[21] Puri S, Chowdhury D and Parekh N 1991 Non-algebraic domain growth in random magnets: a cell dynamical approach *J. Phys. A: Math. Gen.* in press